

1. The equation  $kx^2 + 4kx + 3 = 0$ , where  $k$  is a constant, has no real roots.

Prove that

$$0 \leq k < \frac{3}{4}$$

(4)

If  $k=0$ , then  $0x^2+4(0)x+3=0$

$$3=0$$

which gives no real roots

Discriminant,  $b^2 - 4ac = (4k)^2 - (4)(k)(3)$

$$\text{So, } (4k)^2 - (4)(k)(3) < 0$$

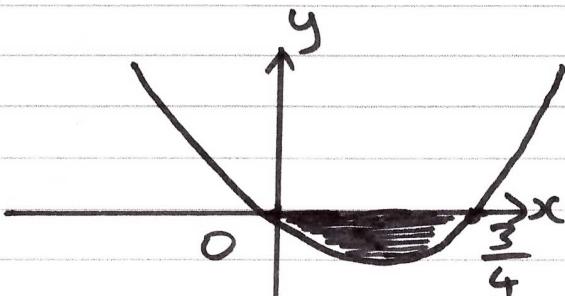
$$16k^2 - 12k < 0$$

For  $16k^2 - 12k = 0$

$$4k(4k-3) = 0$$

Either  $k=0$  or  $k = \frac{3}{4}$

$$0 < k < \frac{3}{4}$$

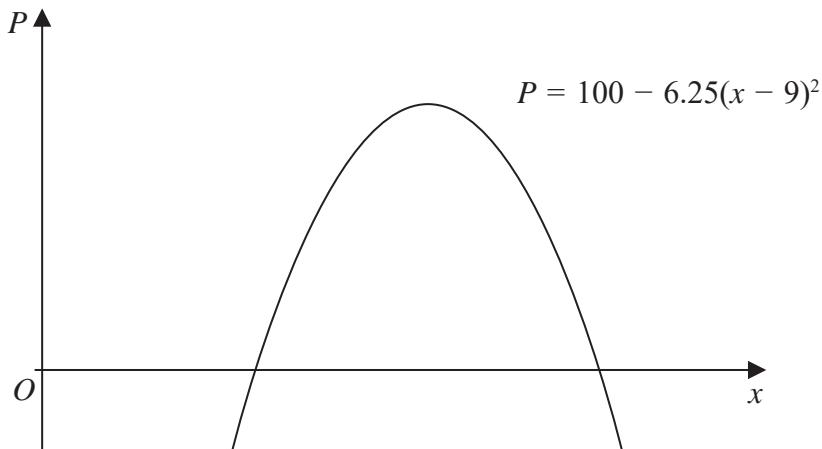


& choose the values 'under' the x-axis, since  $16k^2 - 12k < 0$

Together with  $k=0$  giving an unreal solution,

$$0 \leq k < \frac{3}{4}$$

2.

**Figure 1**

A company makes a particular type of children's toy.

The annual profit made by the company is modelled by the equation

$$P = 100 - 6.25(x - 9)^2$$

where  $P$  is the profit measured in thousands of pounds and  $x$  is the selling price of the toy in pounds.

A sketch of  $P$  against  $x$  is shown in Figure 1.

Using the model,

(a) explain why £15 is not a sensible selling price for the toy. (2)

Given that the company made an annual profit of more than £80 000

(b) find, according to the model, the least possible selling price for the toy. (3)

The company wishes to maximise its annual profit.

State, according to the model,

(c) (i) the maximum possible annual profit,  
(ii) the selling price of the toy that maximises the annual profit. (2)

a)  $P = 100 - 6.25(15-9)^2$

$= -125$

negative profit so company will make a loss

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b)  $P > 80$

$$100 - 6 \cdot 25(n-9)^2 > 80$$

$$(n-9)^2 > 3.2$$

$$n > 9 + \frac{4\sqrt{5}}{5}, n > 9 - \frac{4\sqrt{5}}{5}$$

$$n > 7.21$$

$\therefore$  min price :  $n = £7.21$

i)  $(n-9) = 0$

$$P = 100$$

Max. profit : £100 000

ii)  $n-9=0$

$$n = 9$$

Selling price : £9

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3. A company started mining tin in Riverdale on 1st January 2019.

A model to find the total mass of tin that will be mined by the company in Riverdale is given by the equation

$$T = 1200 - 3(n - 20)^2$$

where  $T$  tonnes is the total mass of tin mined in the  $n$  years after the start of mining.

Using this model,

- (a) calculate the mass of tin that will be mined up to 1st January 2020,

(1)

- (b) deduce the maximum total mass of tin that could be mined,

(1)

- (c) calculate the mass of tin that will be mined in 2023.

(2)

- (d) State, giving reasons, the limitation on the values of  $n$ .

(2)

a)  $n = 1$  :  $T = 1200 - 3(1-20)^2$

$$= \boxed{117} \text{ tonnes}$$

b)  $1200$ . (when  $T = 20$  the  $2^{\text{nd}}$  term disappears)

c)  $T_5 - T_4 = [1200 - 3(5-20)^2] - [1200 - 3(4-20)^2]$

$$= \boxed{93} \text{ tonnes}$$

d) As observed in part (b),  $T_{\max}$  is reached when  $n = 20$ , hence the model is only valid for  $n \leq 20$ .

The total mass of tin cannot decrease.



4.

In this question you should show all stages of your working.

Solutions relying on calculator technology are not acceptable.

- (a) Using algebra, find all solutions of the equation

$$3x^3 - 17x^2 - 6x = 0 \quad (3)$$

- (b) Hence find all real solutions of

$$3(y-2)^6 - 17(y-2)^4 - 6(y-2)^2 = 0 \quad (3)$$

$$\begin{aligned} (a) \quad & 3x^3 - 17x^2 - 6x = 0 \\ & x(3x^2 - 17x - 6) = 0 \quad (1) \\ & x(3x+1)(x-6) = 0 \quad (1) \end{aligned}$$

$$\begin{aligned} x = 0 \quad \text{or} \quad 3x+1 = 0 \quad \text{or} \quad x-6 = 0 \\ x = -\frac{1}{3} \quad \quad \quad x = 6 \end{aligned}$$

$$\therefore x = 0, -\frac{1}{3}, 6 \quad * \quad (1)$$

$$(b) \quad 3(y-2)^6 - 17(y-2)^4 - 6(y-2)^2 = 0$$

convert this to the  
equation in (a)

$$3((y-2)^2)^3 - 17((y-2)^2)^2 - 6((y-2)^2) = 0$$

By comparing the equation to (a), we can conclude that  $x = (y-2)^2 \quad (1)$

$$(y-2)^2 = 0, -\frac{1}{3}, 6 \rightarrow \text{use real solutions from (a)}$$

$$\begin{aligned} (y-2)^2 = 0 \quad \text{or} \quad (y-2)^2 = -\frac{1}{3} \quad \text{or} \quad (y-2)^2 = 6 \\ y-2 = 0 \quad \quad \quad \therefore \text{no square root} \quad \quad \quad y-2 = \sqrt{6} \\ y = 2 \quad \quad \quad \text{for negative numbers,} \quad \quad \quad y = 2 \pm \sqrt{6} \\ & \quad \quad \quad \text{hence this has no} \\ & \quad \quad \quad \text{real solutions} \end{aligned}$$

$$\therefore \text{real solutions of } y : 2, 2 \pm \sqrt{6} \quad * \quad (1)$$

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5. A curve  $C$  has equation  $y = f(x)$  where

$$f(x) = -3x^2 + 12x + 8$$

- (a) Write  $f(x)$  in the form

$$a(x + b)^2 + c$$

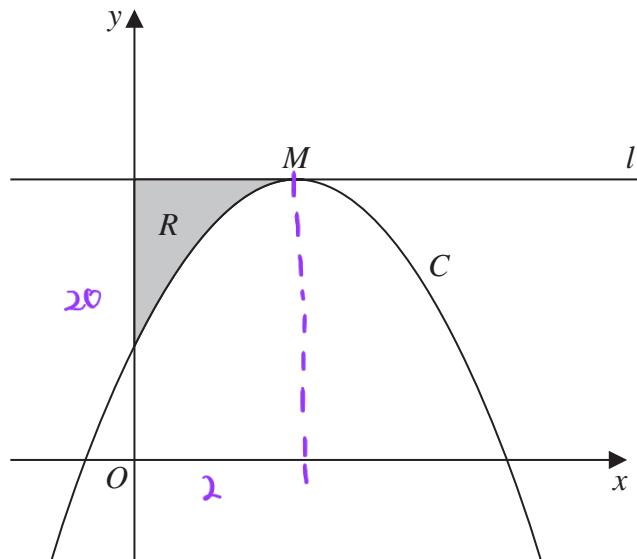
where  $a$ ,  $b$  and  $c$  are constants to be found.

(3)

The curve  $C$  has a maximum turning point at  $M$ .

- (b) Find the coordinates of  $M$ .

(2)



**Figure 3**

Figure 3 shows a sketch of the curve  $C$ .

The line  $l$  passes through  $M$  and is parallel to the  $x$ -axis.

The region  $R$ , shown shaded in Figure 3, is bounded by  $C$ ,  $l$  and the  $y$ -axis.

- (c) Using algebraic integration, find the area of  $R$ .

$$(a) -3x^2 + 12x + 8 \equiv -3(x^2 - 4x) + 8 \quad (1) \quad (5)$$

$$\equiv -3 \left[ \left( x + \frac{-4}{2} \right)^2 - \left( \frac{-4}{2} \right)^2 \right] + 8$$

$$\equiv -3 \left[ (x - 2)^2 - 4 \right] + 8 \quad (1)$$

$$\equiv -3(x - 2)^2 + 12 + 8$$

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$$\equiv -3(x-2)^2 + 20$$

$$\therefore f(x) = -3(x-2)^2 + 20 \quad \text{**} \quad (1)$$

(b) M coordinates can be found using the answer in (a) as M is also the turning point of the curve.

$\therefore$  Hence, M coordinates are  $(2, 20)$  (1)

(c) l intersects y-axis at 20.

$(-b, c)$  from answer in (a).

$$\begin{aligned}\text{Area of rectangle} &= 2 \times 20 \\ &= 40 \text{ units}^2\end{aligned}$$

$$\text{Area under the curve} = \int_0^2 (-3x^2 + 12x + 8) dx \quad (1)$$

$$= \left[ -\frac{3x^3}{3} + \frac{12x^2}{2} + 8x \right]_0^2$$

$$= \left[ -x^3 + 6x^2 + 8x \right]_0^2 \quad (1)$$

$$= \left[ -(2)^3 + 6(2)^2 + 8(2) \right] - 0 \quad (1)$$

$$= [-8 + 24 + 16]$$

$$= 32 \text{ units}^2$$

$$\text{Area of R} = \text{Area of rectangle} - \text{area under the curve} \quad (1)$$

$$= 40 - 32$$

$$\text{Area of R} = 8 \text{ units}^2 \quad \text{**} \quad (1)$$



**6.** An archer shoots an arrow.

The height,  $H$  metres, of the arrow above the ground is modelled by the formula

$$H = 1.8 + 0.4d - 0.002d^2, \quad d \geq 0$$

where  $d$  is the horizontal distance of the arrow from the archer, measured in metres.

Given that the arrow travels in a vertical plane until it hits the ground,

- (a) find the horizontal distance travelled by the arrow, as given by this model.

(3)

- (b) With reference to the model, interpret the significance of the constant 1.8 in the formula.

(1)

- (c) Write  $1.8 + 0.4d - 0.002d^2$  in the form

$$A - B(d - C)^2$$

where  $A$ ,  $B$  and  $C$  are constants to be found.

(3)

It is decided that the model should be adapted for a different archer.

The adapted formula for this archer is

$$H = 2.1 + 0.4d - 0.002d^2, \quad d \geq 0$$

Hence or otherwise, find, for the adapted model

- (d) (i) the maximum height of the arrow above the ground.

- (ii) the horizontal distance, from the archer, of the arrow when it is at its maximum height.

(2)

a)  $H = 1.8 + 0.4d - 0.002d^2$

$$H = 0 \Rightarrow -0.002d^2 + 0.4d + 1.8 = 0 \quad \textcircled{1}$$

Quadratic Formula :  $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$        $a = -0.002, b = 0.4, c = 1.8$

$$\Rightarrow \frac{-0.4 \pm \sqrt{(0.4)^2 - 4(-0.002)(1.8)}}{2(-0.002)} \quad \textcircled{1} \quad \begin{array}{ll} \Rightarrow \text{+ve} & \text{-ve} \\ d = -4.403 & d = 204.403 \\ \Rightarrow \text{Not Valid} & \Rightarrow d = \underline{\underline{204 \text{ m}}} \\ \text{Since } d < 0. & \end{array}$$

b)

$$H = 1.8 + 0.4d - 0.002d^2.$$

$d = 0 \Rightarrow H = 1.8 \Rightarrow 1.8$  is the initial height of the arrow above the ground. (1)

c)  $1.8 + 0.4d - 0.002d^2$

$$\Rightarrow -0.002(d^2 - 200d) + 1.8 \quad (1) \quad * (d-100)^2 - 10,000$$

$$\Rightarrow -0.002 \left[ (d-100)^2 - 10,000 \right] + 1.8 \quad (1)$$

$$\Rightarrow -0.002(d-100)^2 + 20 + 1.8 \Rightarrow A = 21.8$$

$$\Rightarrow -0.002(d-100)^2 + 21.8 \quad (1) \quad B = 0.002$$

$$C = \underline{\underline{100}}$$

d i)  $H = 2.1 + 0.4d - 0.002d^2$

$$\Rightarrow Y\text{-Coordinate of the turning} = 2.1 + 20$$

$$\Rightarrow \text{Max Height} = \underline{\underline{22.1m}} \quad (1)$$

previously  $H = 1.8 + 0.4d - 0.002d^2$

and  $H = -0.002(d-100)^2 + 21.8$

$\Rightarrow \text{Turning Point}$   
 $(100, 21.8)$

d ii)  $2.1 + 0.4d - 0.002d^2 = 22.1$

$$\Rightarrow -0.002d^2 + 0.4d - 20 = 0$$

$$\Rightarrow \frac{-0.4 \pm \sqrt{(0.4)^2 - 4(-0.002)(-20)}}{2(-0.002)} = \begin{array}{ll} +ve & -ve \\ d = 100 & d = 100 \end{array}$$

$$\Rightarrow d = \underline{\underline{100m}}$$

7.

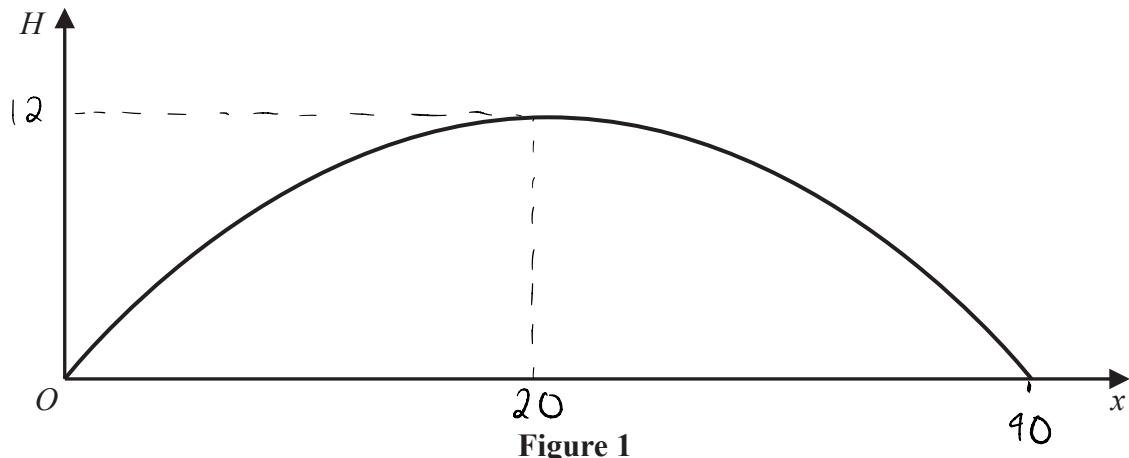


Figure 1 is a graph showing the trajectory of a rugby ball.

The height of the ball above the ground,  $H$  metres, has been plotted against the horizontal distance,  $x$  metres, measured from the point where the ball was kicked.

The ball travels in a vertical plane.

The ball reaches a maximum height of 12 metres and hits the ground at a point 40 metres from where it was kicked.

(a) Find a quadratic equation linking  $H$  with  $x$  that models this situation.

(3)

The ball passes over the horizontal bar of a set of rugby posts that is perpendicular to the path of the ball. The bar is 3 metres above the ground.

(b) Use your equation to find the greatest horizontal distance of the bar from  $O$ .

(3)

(c) Give one limitation of the model.

(1)

$$a) H = ax^2 + bx + c$$

$$(0, 0)$$

$$(20, 12)$$

$$(40, 0)$$

$$0 = a(0)^2 + b(0) + c \rightarrow c = 0$$

$$12 = a(20)^2 + b(20)$$

$$0 = a(40)^2 + b(40)$$

$$12 = 400a + 20b$$

$$0 = 1600a + 40b$$

$$12 = 400(-\frac{1}{40}b) + 20b$$

$$1600a = -40b$$

$$12 = -10b + 20b$$

$$a = -\frac{1}{40}b$$

$$12 = 10b$$

$$a = -\frac{1}{40} \times 1.2 = -0.03$$

$$b = 1.2$$

$$H = -0.03x^2 + 1.2x = \underline{\underline{x(-0.03x + 1.2)}}$$

The ball passes over the horizontal bar of a set of rugby posts that is perpendicular to the path of the ball. The bar is 3 metres above the ground.

(b) Use your equation to find the greatest horizontal distance of the bar from  $O$ . (3)

$$b) H = -0.03x^2 + 1.2x$$

$$3 = -0.03x^2 + 1.2x$$

$$0.03 = \frac{3}{100}$$

$$0.03x^2 - 1.2x + 3 = 0$$

$$x^2 - 40x + 100 = 0 \quad \checkmark$$

$$(x - 20)^2 - 20^2 + 100 = 0$$

$$(x - 20)^2 = 300$$

$$x - 20 = \pm \sqrt{300}$$

$$x = 20 \pm \sqrt{300} \quad \checkmark$$

$$20 + \sqrt{300} > 20 - \sqrt{300}$$

$$x = 20 + \sqrt{300}$$

$$= \underline{\underline{37.3m}} \quad (1 \text{ d.p.}) \quad \checkmark$$

(c) Give one limitation of the model.

(1)

c) There is no modelling of air resistance on the ball. ✓

$$f(x) = 2x^2 + 4x + 9 \quad x \in \mathbb{R}$$

8. (a) Write  $f(x)$  in the form  $a(x + b)^2 + c$ , where  $a$ ,  $b$  and  $c$  are integers to be found.

(3)

- (b) Sketch the curve with equation  $y = f(x)$  showing any points of intersection with the coordinate axes and the coordinates of any turning point.

(3)

- (c) (i) Describe fully the transformation that maps the curve with equation  $y = f(x)$  onto the curve with equation  $y = g(x)$  where

$$g(x) = 2(x - 2)^2 + 4x - 3 \quad x \in \mathbb{R}$$

- (ii) Find the range of the function

$$h(x) = \frac{21}{2x^2 + 4x + 9} \quad x \in \mathbb{R} \quad (4)$$

$$\begin{aligned} a) \quad g(x) &= 2x^2 + 4x + 9 \\ &= 2(x^2 + 2x) + 9 \quad (1) \\ &= 2[(x+1)^2 - 1] + 9 \quad (1) \\ &= 2(x+1)^2 - 2 + 9 \\ &= 2(x+1)^2 + 7 \quad (1) \end{aligned}$$

$$\begin{aligned} b) \quad y &= 2(x+1)^2 + 7 \\ y \text{ intercept when } x &= 0 \\ y &= 2(0+1)^2 + 7 \\ &= 2 + 7 \\ &= 9 \quad \therefore y \text{ intercept } (0, 9) \end{aligned}$$

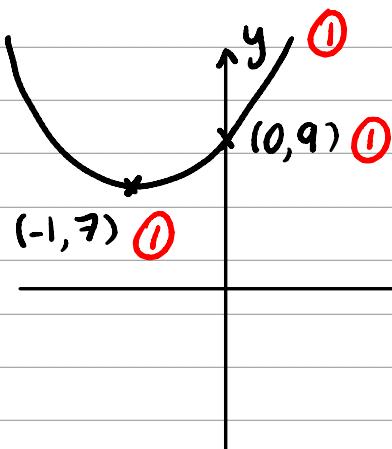
$x$  intercept when  $y = 0$

$$0 = 2(x+1)^2 + 7$$

$$2(x+1)^2 = -7$$

$$(x+1)^2 = \frac{-7}{2} \quad \leftarrow \text{curve doesn't intercept } x \text{ axis}$$

(no real roots)



Turning Point

$$y = (x-a)^2 + b \quad \text{turning point } (a, b)$$

$$y = 2(x-(-1))^2 + 7 \quad \therefore \text{turning point } (-1, 7)$$

c) i)  $f(x) = 2x^2 + 4x + 9$   
 $g(x) = 2(x-2)^2 + 4x - 3$

$f(x-a)$  translation by vector  $\begin{pmatrix} a \\ 0 \end{pmatrix}$

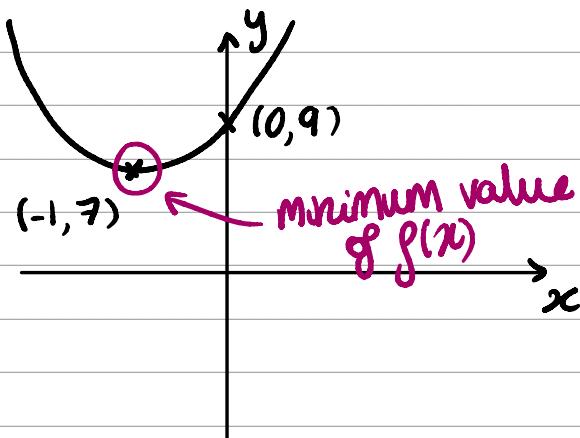
$$\begin{aligned} f(x-2) &= 2(x-2)^2 + 4(x-2) + 9 \\ &= 2(x-2)^2 + 4x - 8 + 9 \\ &= 2(x-2)^2 + 4x + 1 \end{aligned}$$

$f(x)+b$  translation by vector  $\begin{pmatrix} 0 \\ b \end{pmatrix}$

$$\begin{aligned} f(x-2) - 4 &= 2(x-2)^2 + 4(x-2) + 9 - 4 \\ &= 2(x-2)^2 + 4x - 8 + 9 - 4 \\ &= 2(x-2)^2 + 4x - 3 \end{aligned}$$

$$g(x) = f(x-2) - 4 \quad \therefore \text{The transformation that maps } y=f(x) \text{ onto } y=g(x) \text{ is a translation by vector } \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

c) ii)



$$\begin{aligned} h(x) &= \frac{21}{2x^2 + 4x + 9} \\ &= \frac{21}{f(x)} \end{aligned}$$

$$f(x) \rightarrow \pm \infty$$

$$h(x) \rightarrow 0$$

$$0 < h(x) \leq 3 \quad \textcircled{1}$$

Maximum  $h(x)$   
 when  $f(x)$  is at its minimum

$$\frac{21}{7} = 3 \quad \textcircled{1}$$

9.

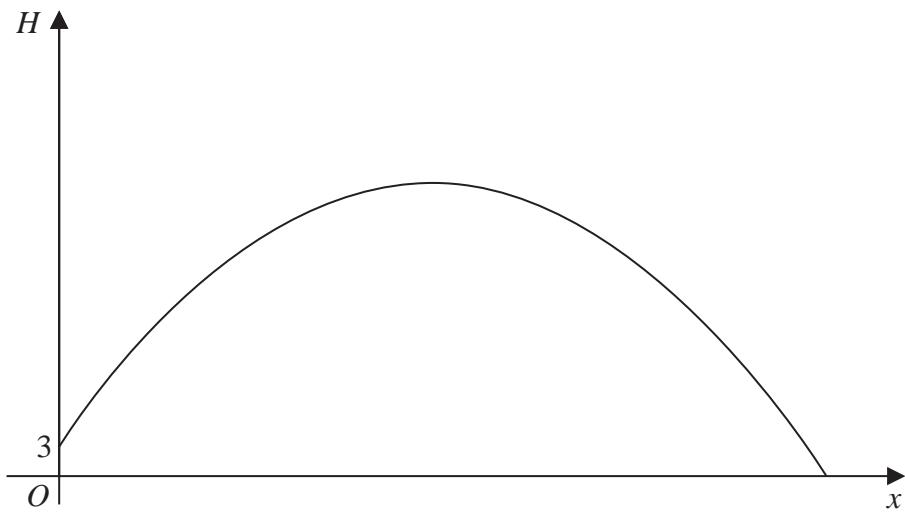
**Figure 3**

Figure 3 is a graph of the trajectory of a golf ball after the ball has been hit until it first hits the ground.

The vertical height,  $H$  metres, of the ball above the ground has been plotted against the horizontal distance travelled,  $x$  metres, measured from where the ball was hit.

The ball is modelled as a particle travelling in a vertical plane above horizontal ground.

Given that the ball

- is hit from a point on the top of a platform of vertical height 3 m above the ground
- reaches its maximum vertical height after travelling a horizontal distance of 90 m
- is at a vertical height of 27 m above the ground after travelling a horizontal distance of 120 m

Given also that  $H$  is modelled as a **quadratic** function in  $x$

(a) find  $H$  in terms of  $x$

(5)

(b) Hence find, according to the model,

- the maximum vertical height of the ball above the ground,
- the horizontal distance travelled by the ball, from when it was hit to when it first hits the ground, giving your answer to the nearest metre.

(3)

(c) The possible effects of wind or air resistance are two limitations of the model.  
Give one other limitation of this model.

(1)



$$(a) H = ax^2 + bx + c$$

$$x=0, H=3 : \quad 3 = 0+0+c \\ c = 3$$

$$H = ax^2 + bx + 3 \quad -\textcircled{1}$$

$$x=120, H=27 : \quad 27 = a(120)^2 + b(120) + 3 \\ 27 = 14400a + 120b + 3 \quad -\textcircled{1} \\ 24 = 14400a + 120b \quad -\textcircled{1}$$

$$\frac{dH}{dx} = 2ax + b \quad -\textcircled{1}$$

$$x=90, \frac{dH}{dx}=0 : \quad 0 = 2a(90) + b \\ = 180a + b \quad -\textcircled{1} \\ b = -180a \quad -\textcircled{2}$$

Substitute  $\textcircled{2}$  into  $\textcircled{1}$

$$24 = 14400a + 120(-180a) \\ = 14400a - 21600a \\ = -7200a$$

$$a = -\frac{1}{300}$$

$$b = -180 \left( -\frac{1}{300} \right)$$

$$= \frac{3}{5}$$

$$\therefore H = -\frac{x^2}{300} + \frac{3x}{5} + 3 \quad * \quad \textcircled{1}$$



$$(b)(i) \quad x = 90 \quad : \quad H = \frac{-(90)^2}{300} + \frac{3(90)}{5} + 3$$

$$= 30 \text{ m} \quad \text{**} \quad (1)$$

$$(ii) \quad H = 0 \quad : \quad 0 = \frac{-x^2}{300} + \frac{3x}{5} + 3 \quad (1)$$

$$x = 184.86\dots, -4.868\dots$$

$$\therefore x = 185 \text{ m} \quad (\text{nearest metre}) \quad (1)$$

only take positive value as the answer

(c) The ground is unlikely to be horizontal (1)

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P 6 8 7 3 1 A 0 4 0 5 2



10. Find, using algebra, all real solutions to the equation

$$(i) \quad 16a^2 = 2\sqrt{a}$$

(4)

$$(ii) \quad b^4 + 7b^2 - 18 = 0$$

(4)

$$i) \quad 16a^2 - 2\sqrt{a} = 0$$

$$a^{\frac{1}{2}}[16a^{\frac{3}{2}} - 2] = 0$$

$$\text{so } a^{\frac{1}{2}} = 0 \quad \text{or} \quad 16a^{\frac{3}{2}} - 2 = 0$$

$$\Rightarrow \boxed{a = 0}$$

$$a^{\frac{3}{2}} = \frac{1}{8}$$

$$a^{\frac{1}{2}} = \frac{1}{2}$$

$$\Rightarrow \boxed{a = \frac{1}{4}}$$

$$ii) \quad \text{let } b^2 = y,$$

$$y^2 + 7y - 18 = 0$$

$$(y+9)(y-2) = 0$$

$$\Rightarrow y = -9$$

$$y = 2$$

$$b^2 = -9$$

$$b^2 = 2$$

[no real  
solutions]

$$\boxed{b = \pm \sqrt{2}}$$



11. The curve  $C$  has equation  $y = f(x)$  where

$$f(x) = ax^3 + 15x^2 - 39x + b$$

and  $a$  and  $b$  are constants.

Given

- the point  $(2, 10)$  lies on  $C$
- the gradient of the curve at  $(2, 10)$  is  $-3$

(a) (i) show that the value of  $a$  is  $-2$

(ii) find the value of  $b$ .

(4)

(b) Hence show that  $C$  has no stationary points.

(3)

(c) Write  $f(x)$  in the form  $(x - 4)Q(x)$  where  $Q(x)$  is a quadratic expression to be found.

(2)

(d) Hence deduce the coordinates of the points of intersection of the curve with equation

$$y = f(0.2x)$$

and the coordinate axes.

(2)

(a) (i)  $f(x) = ax^3 + 15x^2 - 39x + b$

$$f'(x) = 3ax^2 + 30x - 39$$

$$\begin{aligned} f'(2) &= 3a(2)^2 + 30(2) - 39 \\ &= 12a + 60 - 39 \end{aligned}$$

$$f'(2) = 12a + 21$$

Since we were given gradient is  $-3$ , substitute this with  $f'(2)$ .

$$f'(2) = 12a + 21$$

$$-3 = 12a + 21 \quad (1)$$

$$-24 = 12a$$

$$a = -2 \quad \text{※ } (1)$$



$$(a)(ii) f(x) = ax^3 + 15x^2 - 39x + b$$

$$= -2x^3 + 15x^2 - 39x + b$$

$$10 = -2(2)^3 + 15(2)^2 - 39(2) + b \quad (1)$$

$$10 = -16 + 60 - 78 + b$$

$$10 = -34 + b$$

$$b = 44 \quad (1)$$

$$(b) f(x) = -2x^3 + 15x^2 - 39x + 44$$

$$\frac{dy}{dx} = -6x^2 + 30x - 39 \quad (1)$$

$$b^2 - 4ac = (30)^2 - 4(-6)(-39)$$

$$= -36 \quad (1)$$

As  $b^2 - 4ac = -36 < 0$ ,  $f'(x) \neq 0$ . This means that  $f'(x)$  has no real roots. Hence,  $f'(x)$  has no turning points. (1)

$$(c) -2x^3 + 15x^2 - 39x + 44 = (x-4)(ax^2 + bx + c)$$

$$= ax^3 + bx^2 + cx - 4ax^2 - 4bx - 4c$$

$$= ax^3 + bx^2 - 4ax^2 + cx - 4bx - 4c$$

$$= ax^3 + (b-4a)x^2 + (c-4b)x - 4c$$

$$(1) a = -2$$

$$(2) b - 4a = 15$$

$$(3) -4c = 44$$

(1)

$$b - 4(-2) = 15$$

$$c = -11$$

$$b + 8 = 15$$

$$b = 7$$

$$\text{Hence, } f(x) = (x-4)(-2x^2 + 7x - 11) \quad (1)$$



(d) The curve intersects the  $y$ -axis when  $x=0$ . So, substitute  $x=0$  into the equation :

$$\begin{aligned}f(0) &= (0-4)(-2(0)^2 + 7(0) - 11) \\&= -4(-11) \\&= 44\end{aligned}$$

Points of intersection with  $y$ -axis =  $(0, 44)$

The curve intersects the  $x$ -axis when  $y=0$ . So :

$$f(x)=0 \rightarrow (x-4)(-2x^2+7x-11)=0$$

solving this using calculator will not give us real roots

$\therefore x=4$  is the only real solution

$$f(x)=0 \rightarrow (4, 0)$$

The question asks for  $f(0.2x)$ .  $f(x)$  to  $f(0.2x)$  has a scalar factor of 5 in the  $x$ -direction. So,  $(4, 0)$  needs to be multiplied by 5

$$(4, 0) \times 5 = (20, 0)$$

$\therefore$  Hence, the points of intersections are  $(0, 44)$  and  $(20, 0)$

where  $f(0.2x)$  intersects  $y$ -axis

where  $f(0.2x)$  intersects  $x$ -axis

